



## EJERCICIOS RESUELTOS POTENCIAS Y RAICES

### 1) Cálculo y reducción

Introduce los factores dentro del radical:

$$\begin{aligned} \text{a) } 2\sqrt[4]{3} & \qquad 2\sqrt[4]{3} = \sqrt[4]{2^4 \cdot 3} = \sqrt[4]{16 \cdot 3} = \sqrt[4]{48} \\ \text{b) } x^2\sqrt[7]{x^3} & \qquad x^2\sqrt[7]{x^3} = \sqrt[7]{(x^2)^7 \cdot x^3} = \sqrt[7]{x^{14} \cdot x^3} = \sqrt[7]{x^{17}} \end{aligned}$$

Extrae los factores del radical:

$$\begin{aligned} \text{a) } \sqrt[4]{128} & \qquad \sqrt[4]{128} = \sqrt[4]{2^7} = 2\sqrt[4]{2^3} = 2\sqrt[4]{8} \\ \text{b) } \sqrt[7]{x^{30}} & \qquad \sqrt[7]{x^{30}} = \sqrt[7]{x^{28+2}} = \sqrt[7]{x^{28} \cdot x^2} = x^4\sqrt[7]{x^2} \end{aligned}$$

Calcular las siguientes raíces:

$$\begin{aligned} \text{a) } \sqrt[5]{1024} & \qquad \sqrt[5]{1024} = \sqrt[5]{2^{10}} = 2^2 = 4 \\ \text{b) } \sqrt[7]{x^{84}} & \qquad \sqrt[7]{x^{84}} = \sqrt[7]{x^{12 \cdot 7}} = \sqrt[7]{(x^{12})^7} = x^{12} \end{aligned}$$

Reduce a índice común

$$\begin{aligned} \text{a) } \sqrt{3}; \sqrt[3]{5} & \qquad \sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8} \quad ; \quad \sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25} \\ \text{b) } \sqrt[4]{x^3}; \sqrt[6]{x^5} & \qquad \sqrt[4]{x^3} = \sqrt[12]{x^9} \quad ; \quad \sqrt[6]{x^5} = \sqrt[12]{x^{10}} \end{aligned}$$

2) Raíz de un cociente, raíz de un producto, raíz de una potencia, raíz de una raíz, simplificación y racionalización.

Escribe con una sola raíz:

$$\begin{aligned} \text{a) } \sqrt[5]{\sqrt{3}} &= \sqrt[10]{3} \\ \text{b) } \sqrt[7]{X^4 \cdot \sqrt{X}} &= \sqrt[7]{\sqrt{X^8 \cdot X}} = \sqrt[14]{X^9} \end{aligned}$$

Escribe con una sola raíz:

$$\begin{aligned} \text{a) } \sqrt[4]{3 \cdot \sqrt[4]{27}} &= \sqrt[4]{3 \cdot 3} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3 \\ \text{b) } \sqrt[5]{X \cdot \sqrt[5]{X^2}} &= \sqrt[5]{X^3} \end{aligned}$$

Escribe con una sola raíz:

$$\begin{aligned} \text{a) } \frac{\sqrt[3]{16}}{\sqrt[3]{2}} &= \sqrt[3]{\frac{16}{2}} = \sqrt[3]{8} = 2 \\ \text{b) } \frac{\sqrt[5]{X^4}}{\sqrt[5]{X^3}} &= \sqrt[5]{\frac{X^4}{X^3}} = \sqrt[5]{X} \end{aligned}$$

Racionaliza.

$$\begin{aligned} \text{a) } \frac{1}{\sqrt[5]{9}} &= \frac{1}{\sqrt[5]{3^2}} = \frac{1 \cdot \sqrt[5]{3^3}}{\sqrt[5]{3^2 \cdot 3^3}} = \frac{\sqrt[5]{3^3}}{\sqrt[5]{3^5}} = \frac{\sqrt[5]{9}}{3} \\ \text{b) } \frac{2}{5 \cdot \sqrt[3]{4}} &= \frac{2}{5 \cdot \sqrt[3]{2^2}} = \frac{2 \cdot \sqrt[3]{2}}{5 \cdot \sqrt[3]{2^2 \cdot 2}} = \frac{2 \cdot \sqrt[3]{2}}{5 \cdot \sqrt[3]{2^3}} = \frac{2 \cdot \sqrt[3]{2}}{5 \cdot 2} = \frac{\sqrt[3]{2}}{5} \end{aligned}$$

Racionaliza:

$$\begin{aligned} \text{a) } \frac{1}{\sqrt[7]{X^4}} &= \frac{1 \cdot \sqrt[7]{X^3}}{\sqrt[7]{X^4 \cdot X^3}} = \frac{\sqrt[7]{X^3}}{\sqrt[7]{X^7}} = \frac{\sqrt[7]{X^3}}{X} \\ \text{b) } \frac{1}{X^2 \sqrt[7]{X^3}} &= \frac{1 \cdot \sqrt[7]{X^4}}{X^2 \sqrt[7]{X^3 \cdot X^4}} = \frac{\sqrt[7]{X^4}}{X^2 \sqrt[7]{X^7}} = \frac{\sqrt[7]{X^4}}{X^2 \cdot X} = \frac{\sqrt[7]{X^4}}{X^3} \end{aligned}$$

Racionaliza:

$$\begin{aligned} \text{a) } \frac{1}{\sqrt{3} - \sqrt{2}} &= \frac{1(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} = \frac{(\sqrt{3} + \sqrt{2})}{3 - 2} = (\sqrt{3} + \sqrt{2}) \\ \text{b) } \frac{\sqrt{2}}{\sqrt{5} + 2} &= \frac{\sqrt{2}(\sqrt{5} - 2)}{(\sqrt{5} + 2)(\sqrt{5} - 2)} = \frac{\sqrt{10} - 2\sqrt{2}}{5 - 4} = \sqrt{10} - 2\sqrt{2} \\ \text{c) } \frac{1}{3 - \sqrt{x}} &= \frac{1(3 + \sqrt{x})}{(3 - \sqrt{x})(3 + \sqrt{x})} = \frac{3 + \sqrt{x}}{9 - x} \end{aligned}$$

### 3) Operaciones

Calcular la suma:

$$\begin{aligned}
 \text{a) } \sqrt{40} + \sqrt{90} &= \sqrt{4 \cdot 10} + \sqrt{9 \cdot 10} = 2\sqrt{10} + 3\sqrt{10} = 5\sqrt{10} \\
 \text{b) } 2\sqrt{32} - \sqrt{8} &= 2\sqrt{2^5} - \sqrt{2^3} = 2 \cdot 2^2\sqrt{2} - 2\sqrt{2} = 8\sqrt{2} - 2\sqrt{2} = 6\sqrt{2} \\
 \text{c) } \sqrt[3]{4} + \sqrt[5]{16} &= \sqrt[3]{4} + \sqrt[5]{4^2} = \sqrt[3]{4} + \sqrt[3]{4} = 2\sqrt[3]{4} \\
 \text{d) } 2\sqrt{\frac{1}{2}} + 5\sqrt{8} &= \sqrt{\frac{4 \cdot 1}{2}} + 5\sqrt{2^3} = \sqrt{2} + 10\sqrt{2} = 12\sqrt{2}
 \end{aligned}$$

Calcular y simplificar:

$$\begin{aligned}
 \text{a) } \sqrt[3]{3} \cdot \sqrt[4]{27} &= \sqrt[3]{3} \cdot \sqrt[4]{3^3} = \sqrt[3]{3^4} = 3 \\
 \text{b) } \sqrt[3]{x} \cdot \sqrt[5]{x^2} &= \sqrt[15]{x^7} \\
 \text{c) } \sqrt[5]{x^3} \sqrt{x} \cdot \sqrt{x} &= \sqrt[5]{x^3 \cdot x \cdot x} = \sqrt[5]{x^5} = x \\
 \text{d) } \sqrt[3]{2} \cdot \sqrt{2} \cdot \sqrt[4]{8} &= \sqrt[3]{2} \cdot \sqrt{2} \cdot \sqrt[4]{2^3} = \sqrt[12]{2^4 \cdot 2^6 \cdot 2^9} = \sqrt[12]{2^{19}} = 2^{\frac{19}{12}}
 \end{aligned}$$

Calcular y simplificar:

$$\begin{aligned}
 \text{a) } \frac{\sqrt[3]{16}}{\sqrt[5]{2}} &= \frac{\sqrt[3]{2^4}}{\sqrt[5]{2}} = \frac{\sqrt[15]{2^{20}}}{\sqrt[15]{2^3}} = \sqrt[15]{2^{17}} = 2^{\frac{17}{15}} = 2^{\frac{17}{15}} \\
 \text{b) } \frac{\sqrt{x^4}}{\sqrt[14]{x^3}} &= \frac{\sqrt[14]{x^8}}{\sqrt[14]{x^3}} = \sqrt[14]{x^5} \\
 \text{a) } \frac{\sqrt[5]{8^4}}{\sqrt[8]{4^3}} &= \frac{\sqrt[5]{(2^3)^4}}{\sqrt[8]{(2^2)^3}} = \frac{\sqrt[5]{2^{12}}}{\sqrt[8]{2^6}} = \frac{\sqrt[24]{(2^{12})^4}}{\sqrt[24]{(2^6)^3}} = \frac{\sqrt[24]{2^{48}}}{\sqrt[24]{2^{18}}} = \sqrt[24]{2^{30}} = \sqrt[24]{2^{30}} = \sqrt[24]{2^{30}} = 2^{\frac{5}{4}} = 2\sqrt[4]{2} \\
 \text{b) } \frac{\sqrt[3]{x^4} \sqrt{x}}{\sqrt{x}} &= \frac{\sqrt[3]{x^4} \sqrt{x}}{\sqrt{x}} = \frac{\sqrt[3]{x^4 \cdot x^8}}{\sqrt{x}} = \frac{\sqrt[3]{x^{12}}}{\sqrt{x}} = \frac{\sqrt[12]{x^{18}}}{\sqrt[12]{x^3}} = \sqrt[12]{x^{15}} = x^{\frac{5}{4}}
 \end{aligned}$$

Calcular y simplificar

$$\begin{aligned}
 \text{a) } \frac{\sqrt{2} \cdot \sqrt[3]{4}}{\sqrt[4]{8}} &= \frac{\sqrt{2} \cdot \sqrt[3]{2^2}}{\sqrt[4]{2^3}} = \frac{\sqrt[12]{2^6 \cdot 2^8}}{\sqrt[12]{2^9}} = \frac{\sqrt[12]{2^{14}}}{\sqrt[12]{2^9}} = \sqrt[12]{2^5} = \sqrt[12]{2^5} = 2^{\frac{5}{12}} \\
 \text{b) } \frac{\sqrt[5]{2\sqrt{2}} \cdot \sqrt[3]{4}}{\sqrt{8}} &= \frac{\sqrt[5]{2 \cdot 2^{\frac{1}{2}}} \cdot \sqrt[3]{2^2}}{\sqrt{2^3}} = \frac{\sqrt[10]{2^3} \cdot \sqrt[3]{2^2}}{\sqrt{2^3}} = \frac{\sqrt[30]{2^9} \cdot \sqrt[30]{2^{20}}}{\sqrt[30]{2^{45}}} = \frac{\sqrt[30]{2^{29}}}{\sqrt[30]{2^{45}}} \\
 &= \frac{1}{\sqrt[30]{2^{16}}} = \frac{\sqrt[30]{2^{14}}}{\sqrt[30]{2^{16} \cdot \sqrt[30]{2^{14}}}} = \frac{\sqrt[30]{2^{14}}}{\sqrt[30]{2^{30}}} = \frac{\sqrt[30]{2^{14}}}{2} = \frac{\sqrt[30]{2^{14}}}{2}
 \end{aligned}$$

#### 4) Otros

$$1 \quad \sqrt{\sqrt[3]{4\sqrt{2}}} =$$

$$\sqrt{\sqrt[3]{4\sqrt{2}}} = \sqrt[24]{2}$$

$$2 \quad \sqrt{2 \sqrt[3]{2 \sqrt[4]{2}}} =$$

$$\sqrt{2 \sqrt[3]{2 \sqrt[4]{2}}} = \sqrt[3]{2^3 \cdot 2 \sqrt[4]{2}} = \sqrt[3]{2^4 \sqrt[4]{2}} =$$

$$= \sqrt[3]{\sqrt[4]{(2^4)^4} \cdot 2} = \sqrt[3]{\sqrt[4]{2^{16}} \cdot 2} = \sqrt[24]{2^{17}}$$

$$3 \quad \sqrt[3]{\sqrt[3]{\sqrt[3]{2\sqrt{2}}}} =$$

$$\sqrt[3]{\sqrt[3]{\sqrt[3]{2\sqrt{2}}}} = \sqrt[3]{\sqrt[3]{\sqrt[3]{2 \cdot 2^2}}} = \sqrt[3]{\sqrt[3]{\sqrt[3]{2^3}}} = \sqrt[72]{2^3} = \sqrt[24]{2}$$

$$4. \quad (\sqrt{7} - \sqrt{2})^2 =$$

$$= (\sqrt{7})^2 - 2 \cdot \sqrt{7} \cdot \sqrt{2} + (\sqrt{2})^2 =$$

$$= 7 - 2\sqrt{14} + 2 = 9 - 2\sqrt{14}$$

$$5. (\sqrt{5} + 2) \cdot (\sqrt{5} - 2) =$$

$$= (\sqrt{5})^2 - 2^2 = 5 - 4 = 1$$

$$6. (2\sqrt{5} + 3\sqrt{2}) \cdot (2\sqrt{5} - 3\sqrt{2}) =$$

$$= (2\sqrt{5})^2 - (3\sqrt{2})^2 =$$

$$= 2^2 \cdot (\sqrt{5})^2 - 3^2 (\sqrt{2})^2 =$$

$$= 4 \cdot 5 - 9 \cdot 2 = 20 - 18 = 2$$

$$7. \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} =$$

$$= \frac{\sqrt{2} \cdot (\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{3} + \sqrt{2})} =$$

$$= \frac{\sqrt{6} + \sqrt{2^2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{2 + \sqrt{6}}{3 - 2} = 2 + \sqrt{6}$$

$$8. \frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} =$$

$$= \frac{(3\sqrt{2} - 2\sqrt{3}) \cdot (3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2} + 2\sqrt{3}) \cdot (3\sqrt{2} - 2\sqrt{3})} =$$

$$= \frac{(3\sqrt{2} - 2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= \frac{(3\sqrt{2})^2 - 2 \cdot 3 \cdot \sqrt{2} \cdot 2\sqrt{3} + (2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= \frac{9 \cdot 2 - 12\sqrt{6} + 4 \cdot 3}{9 \cdot 2 - 4 \cdot 3} =$$

$$= \frac{18 - 12\sqrt{6} + 12}{18 - 12} =$$

$$= \frac{30 - 12\sqrt{6}}{6} = 5 - 2\sqrt{6}$$